# Pattern Recognition in Almost Empty Spaces

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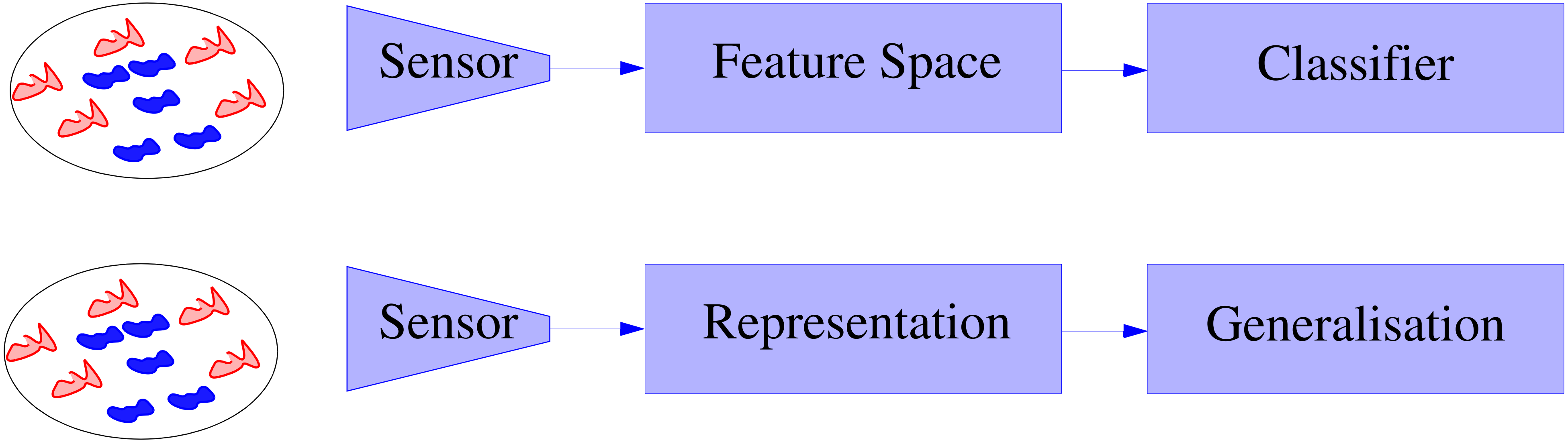
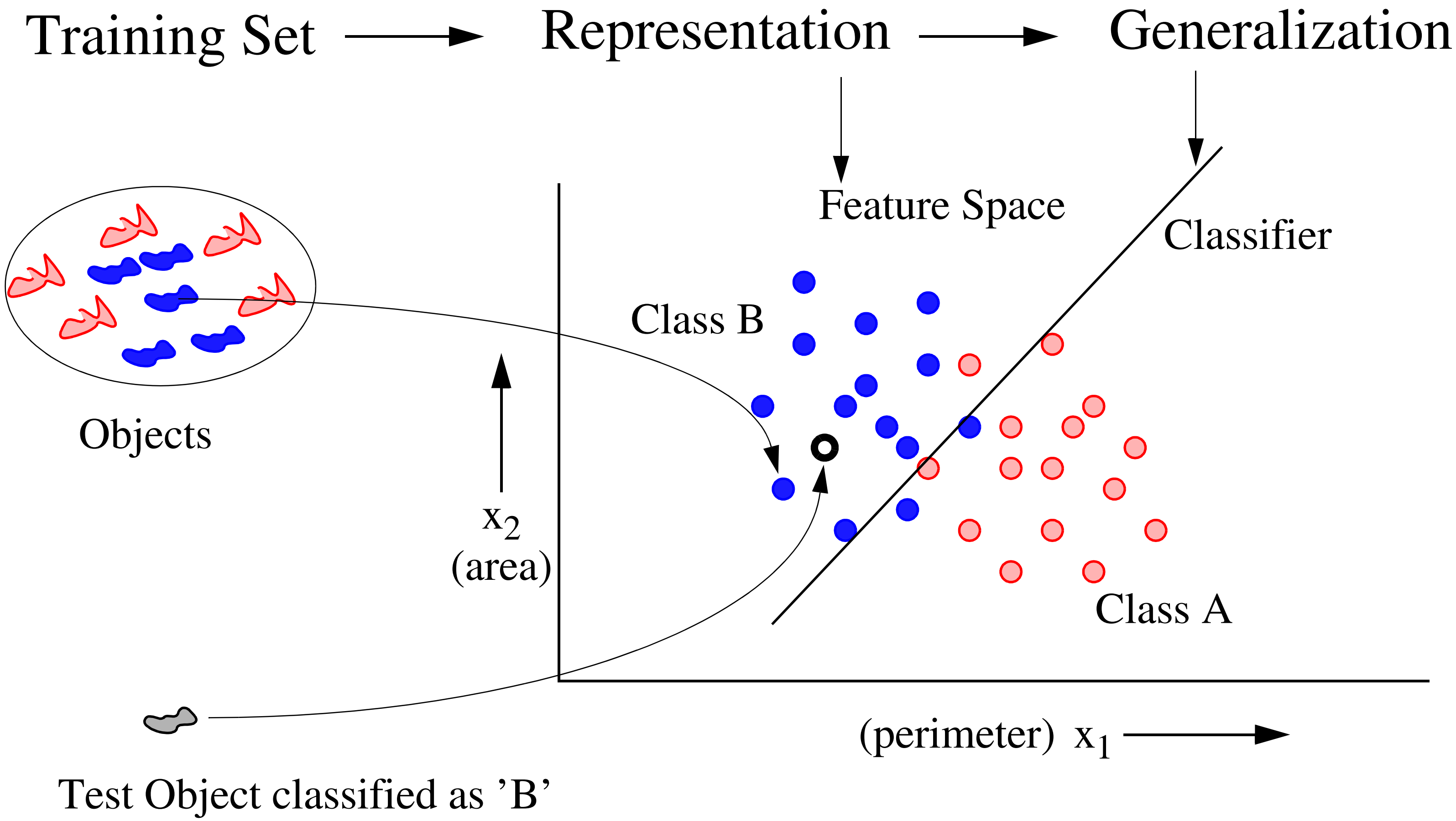
- Pseudo Fisher Linear Discriminant Experiments
- Representation Sets and Kernel Mapping
- Support Vector Classifier
- Dissimilarity Based Classifier
- Subspace Classifier
- Conclusions

# What is Pattern Recognition?




Other representations and generalisations?

# Pattern Recognition System



# Statistical PR Paradox

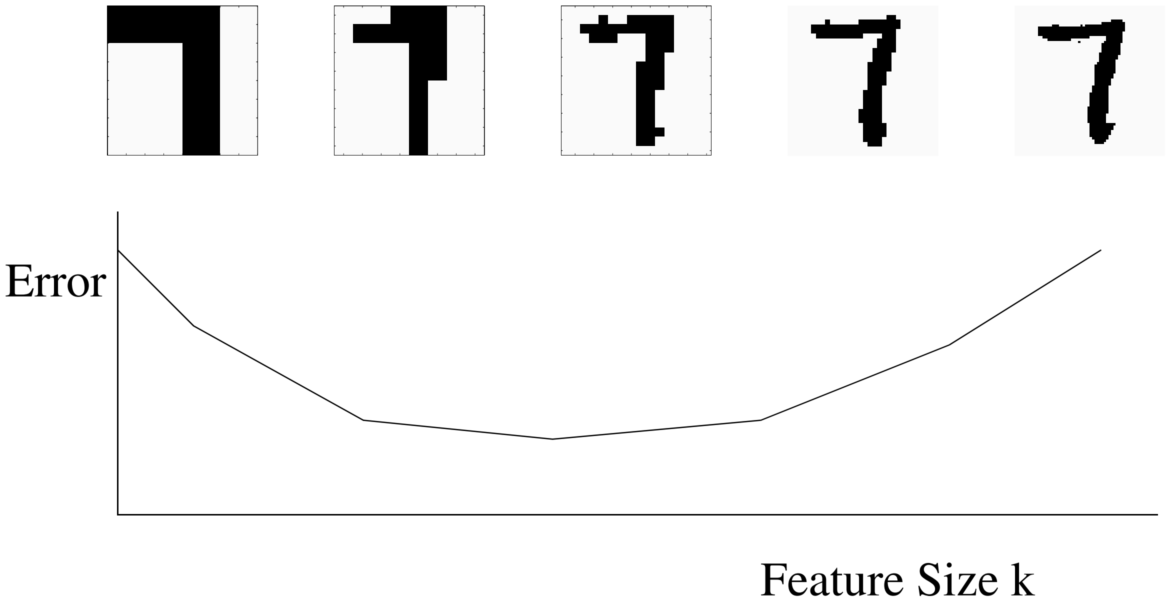
$\mathbf{x} = (x^1, x^2, \ldots , x^k)$ - k dimensional feature space

$\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m\}$ - training set

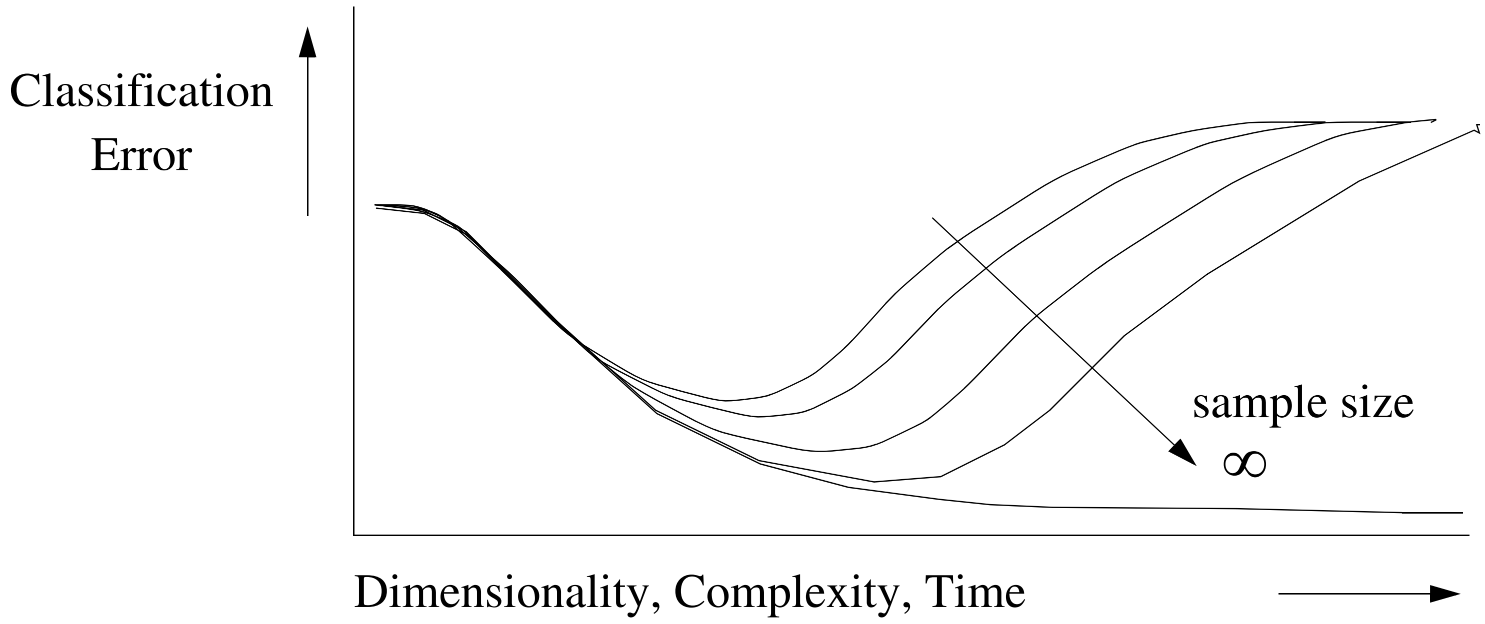
$\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$ - class labels

$\Big\}$ $D(\mathbf{x})$ - classifier , $\varepsilon = \text{Prob}\,(\, D(\mathbf{x}) \neq \lambda(\mathbf{x})\,)$

$\varepsilon(m)$ : monotonically decreasing , $\varepsilon(k)$ : peaks !



# Peaking, Curse of Dimensionality, Overtraining

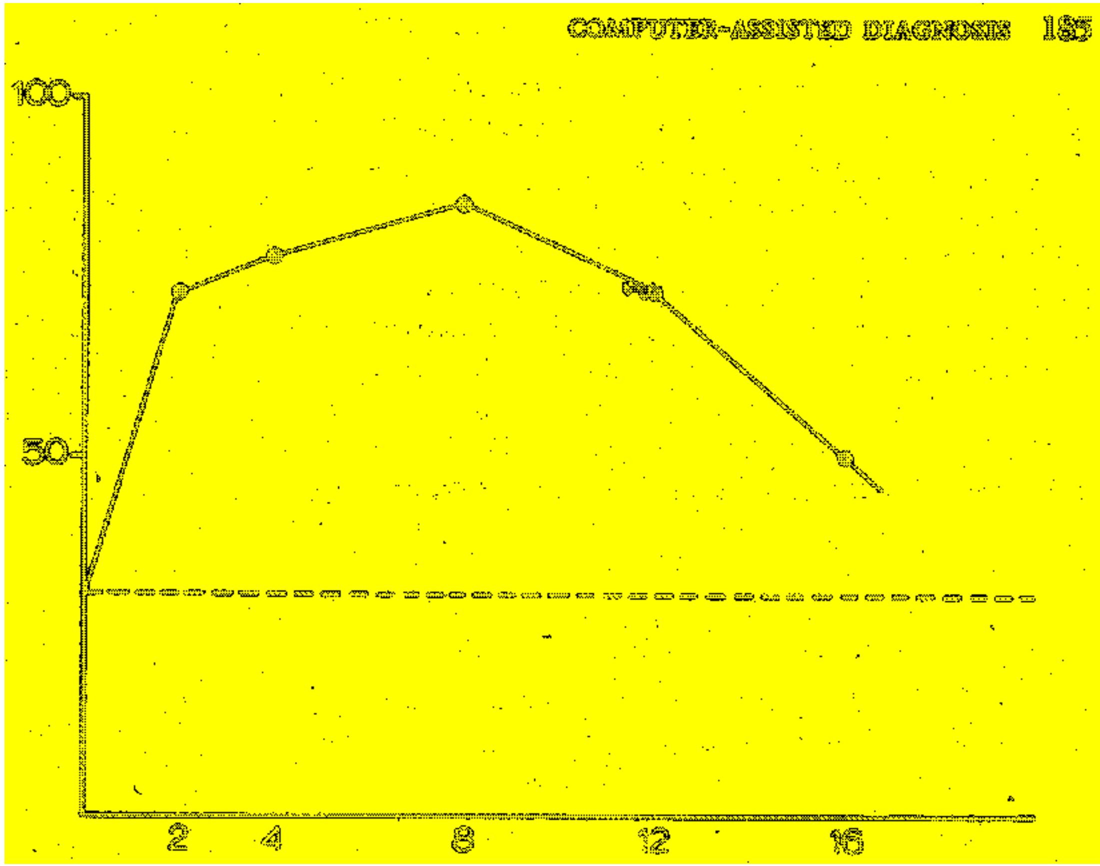



Asymptotically increasing classification error due to:

- Increasing Dimensionality — *Curse of Dimensionality*
- Increasing Complexity — *Peaking Phenomenon*
- Decreasing Regularization
- Increasing Computational Effort

} *Overtraining*

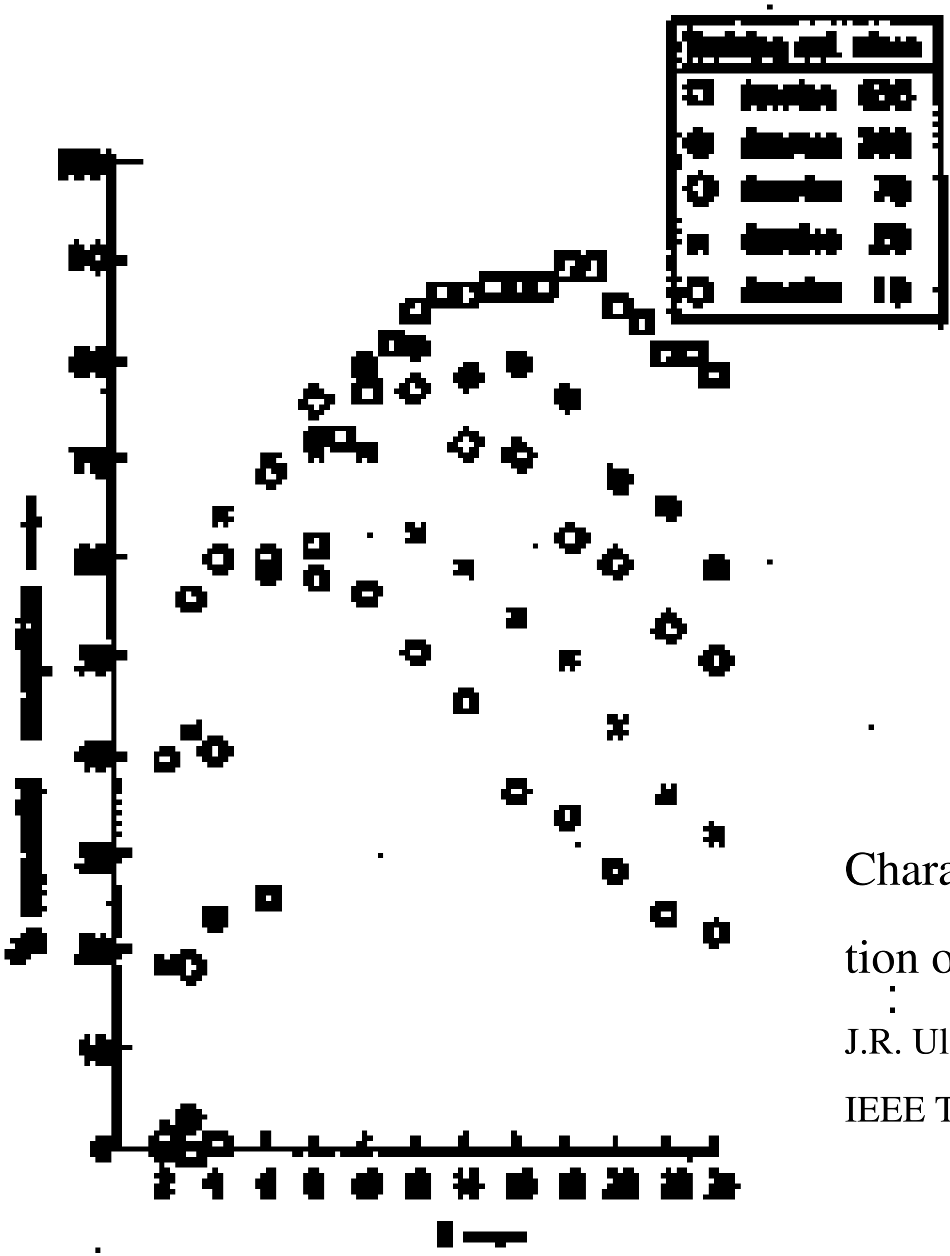
# Human Recognition Accuracy




The diagnostic classification accuracy of a group of doctors for an increasing set of symptons. Samples size is 100.

F.T de Dombal, Computer-assisted diagnosis, in: Principals and practice of medical computing, Whitby and Lutz (eds.), Churchill Livingstone, London, 1971, pp 179 - 199

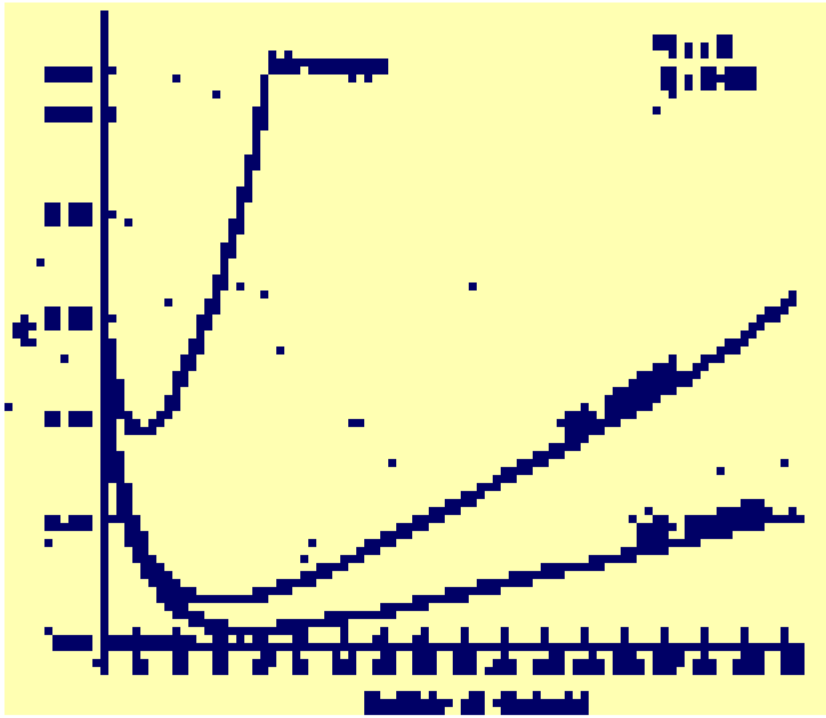
# Ullman's Example



Character recognition classification performance as a function of the number of n-tuples used.

J.R. Ullmann, Experiments with the n-tuple method of pattern recognition, IEEE Trans. on Computers, 1969, 1135-1136

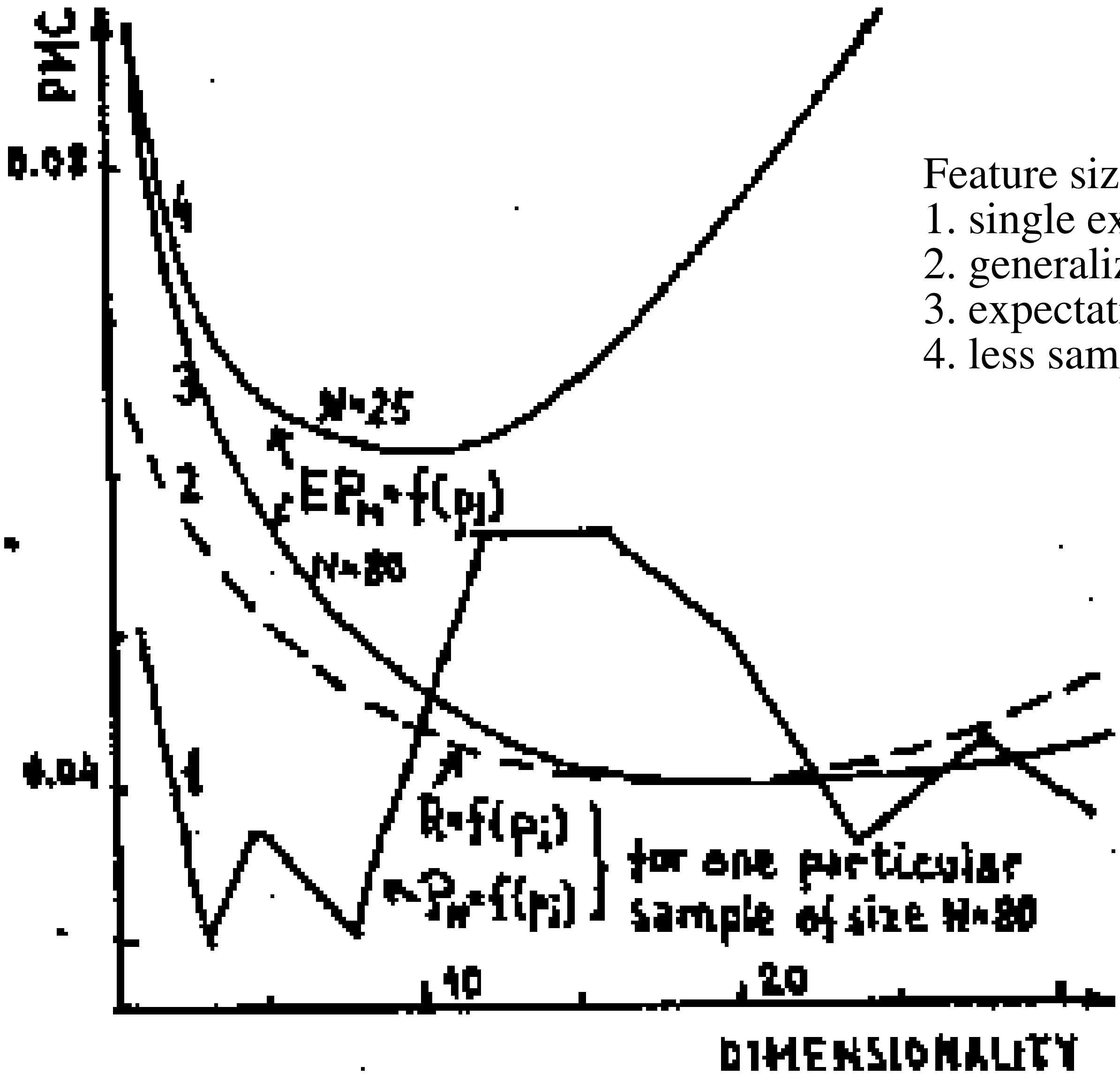
# Simulated Peaking Phenomenon by Jain and Waller



Classification error as a function of the feature size for two overlapping Gaussian distributions. Higher features have increasing class overlap.

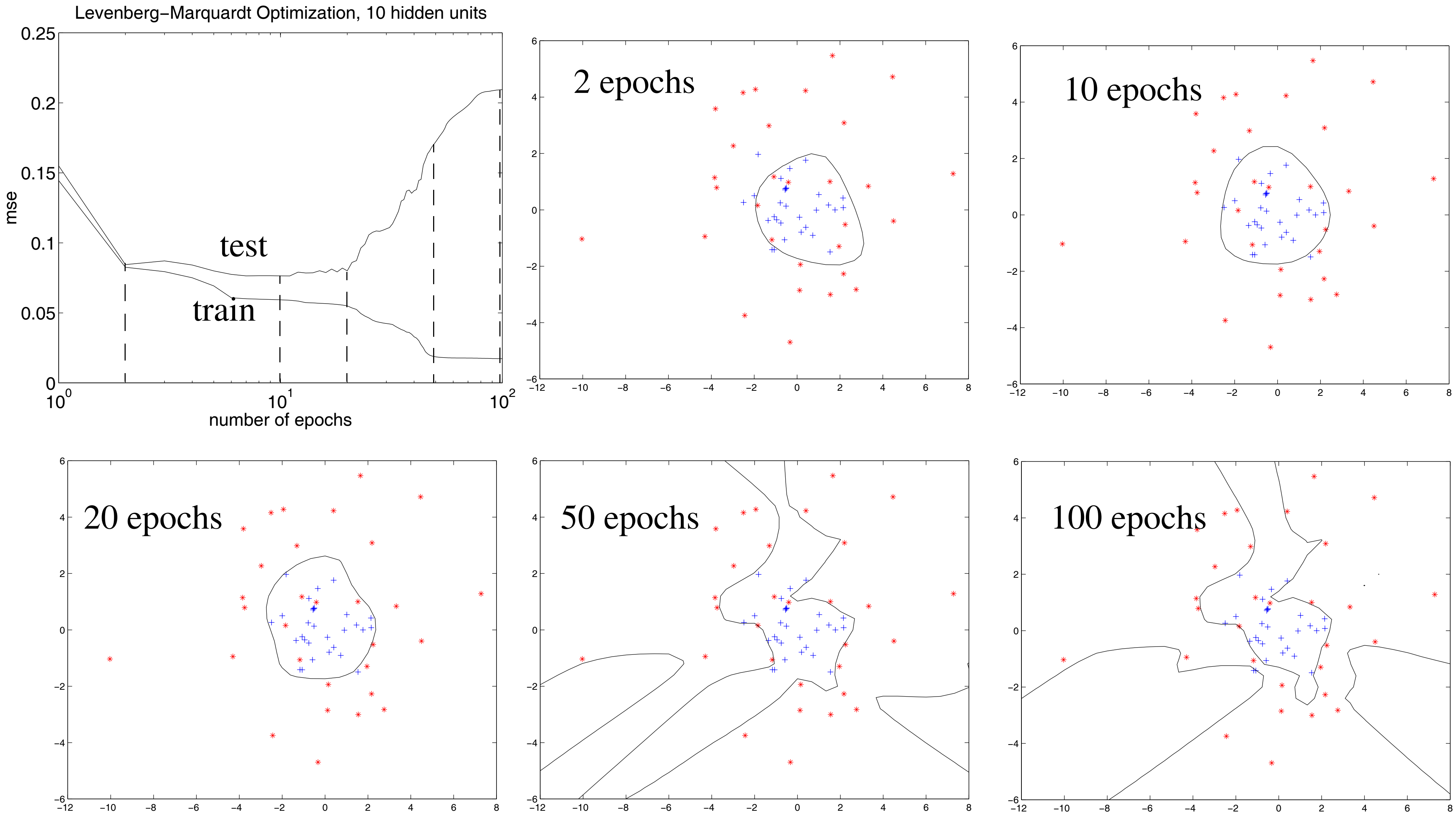
A.K Jain and W.G. Waller, On the optimal number of features in the classification of multivariate Gaussian data, Pattern Recognition, 10, pp 365 - 374, 1978

# Raudys’ Example



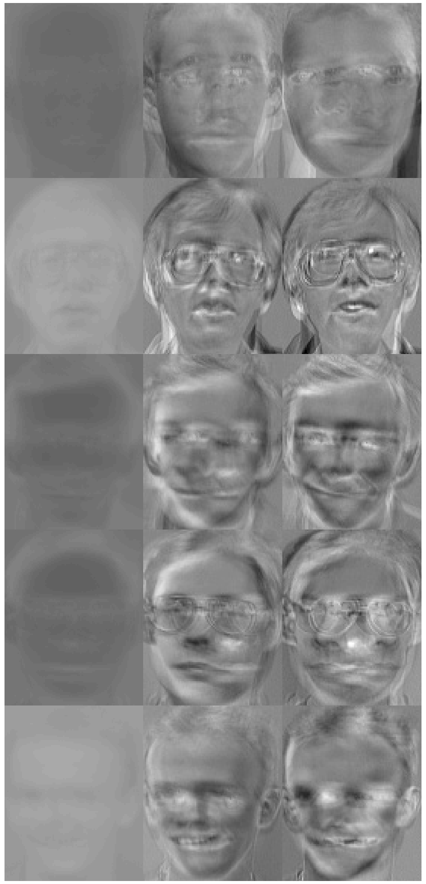

Feature size study by Raudys (3rd ICPR 1976):
1. single example
2. generalization of 1
3. expectation of 1
4. less samples

# Neural Network Overtraining Example - 10 Hidden Units



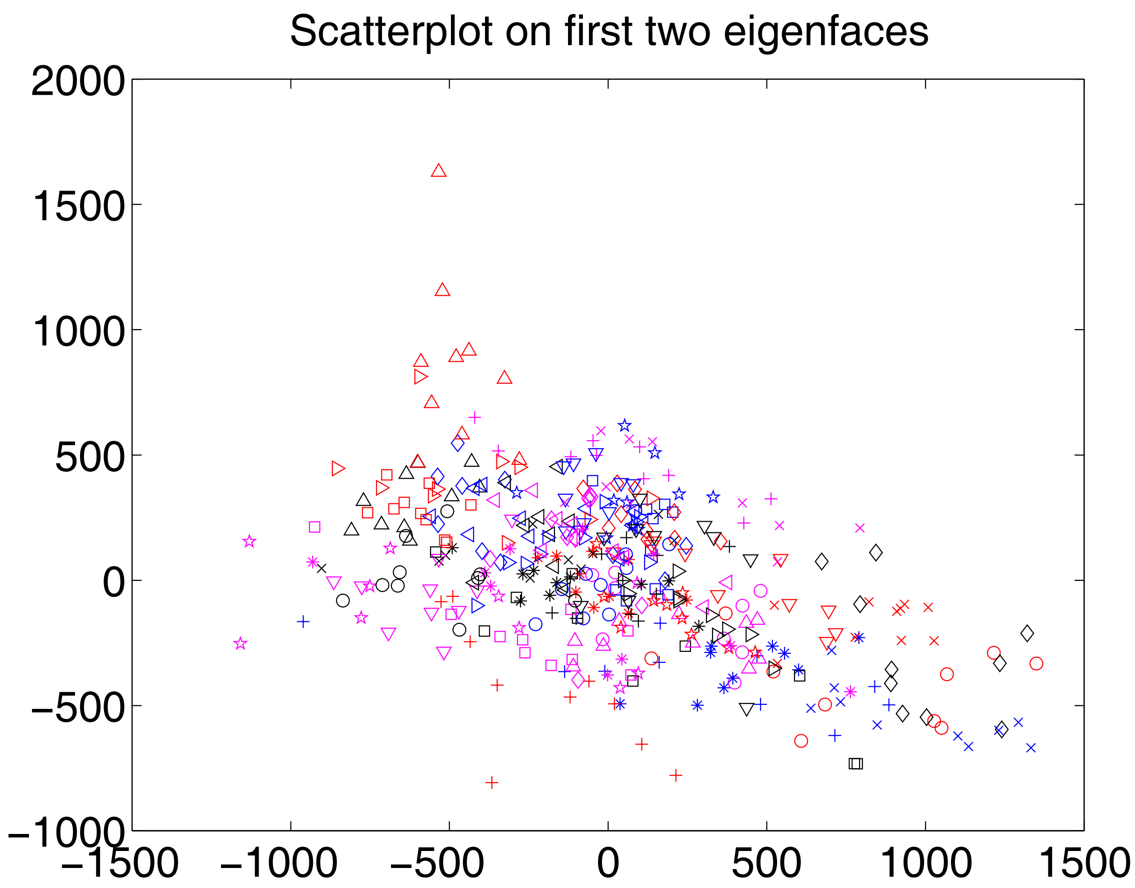
# Eigenfaces

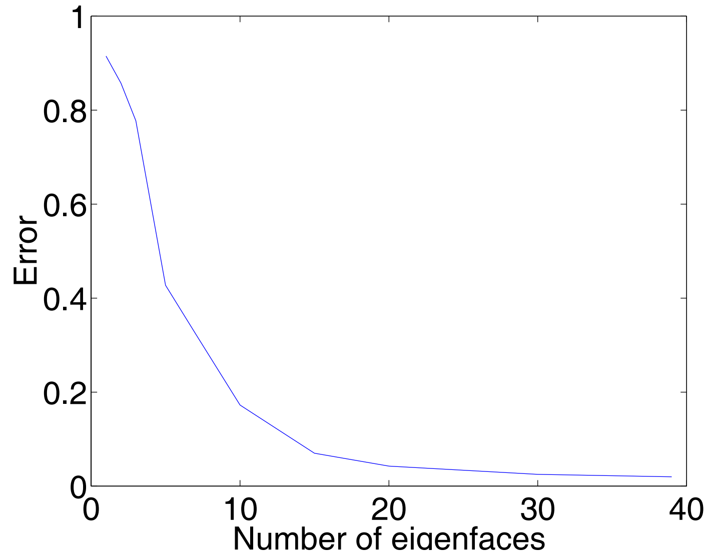
10 pictures of 5 subjects



eigenfaces 1 - 3

# PCA Classification of Faces



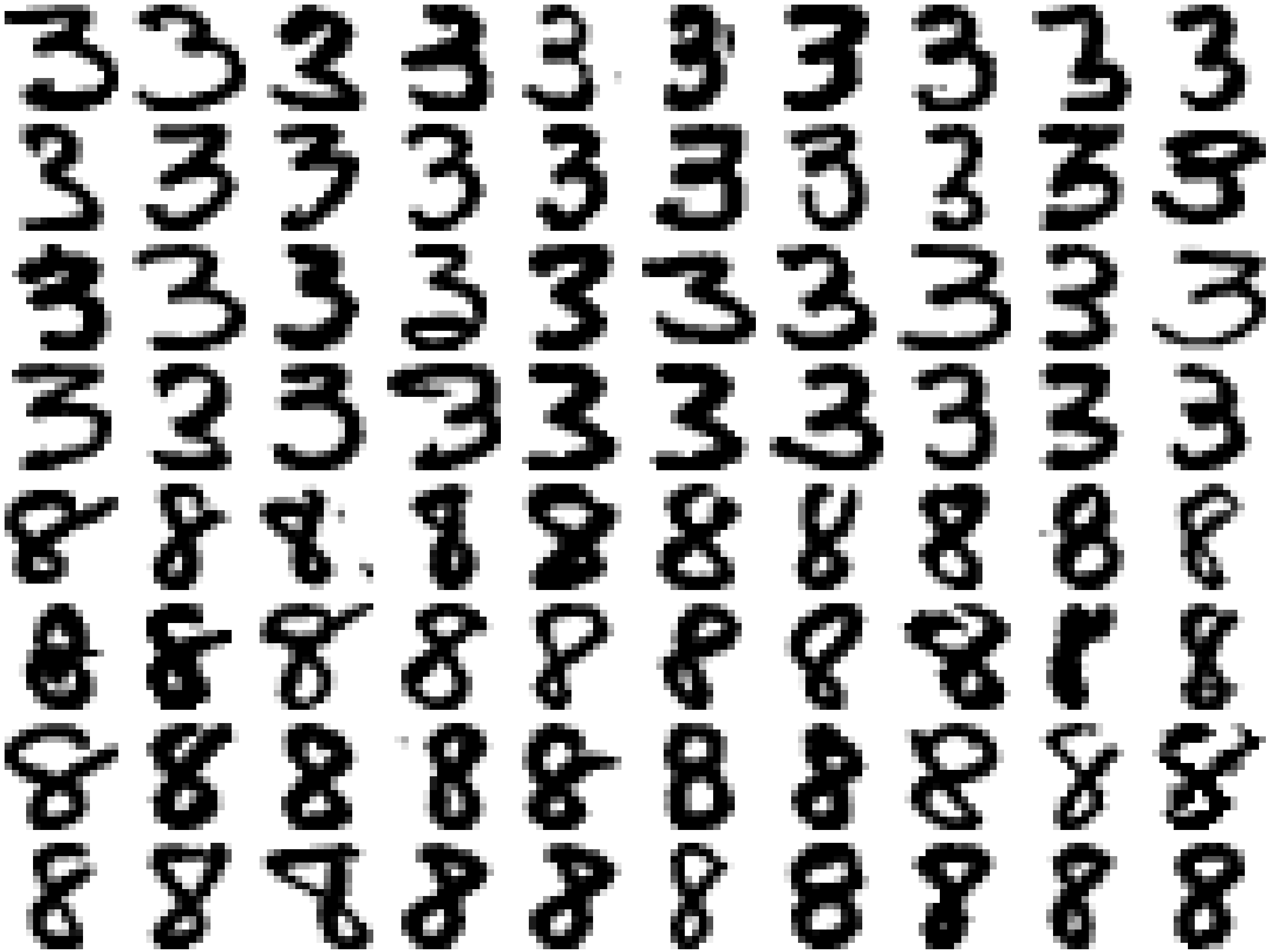




Training Set: 1 image, 40 persons

Feature Size: 92 x 112 = 10304

Test Set: 9 * 40 = 360;

# Normalized NIST Data



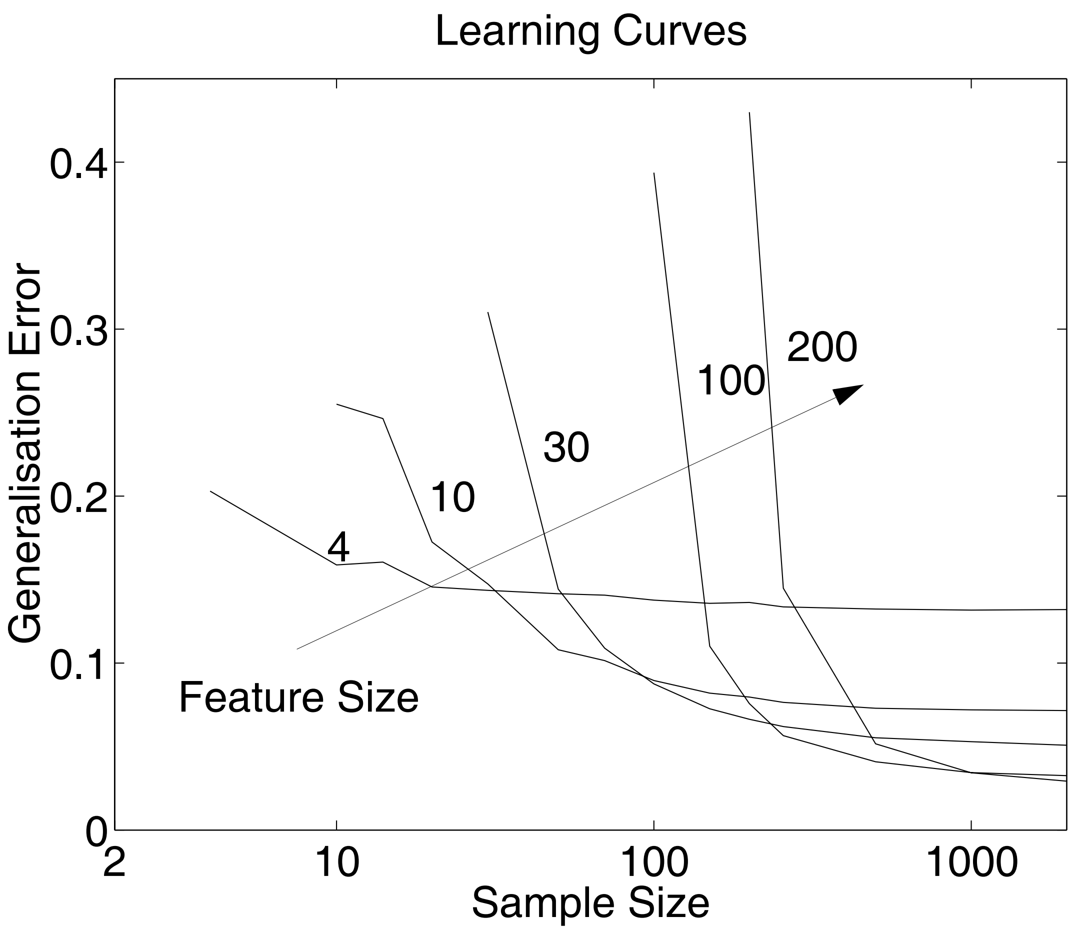
2 x 2000 Characters

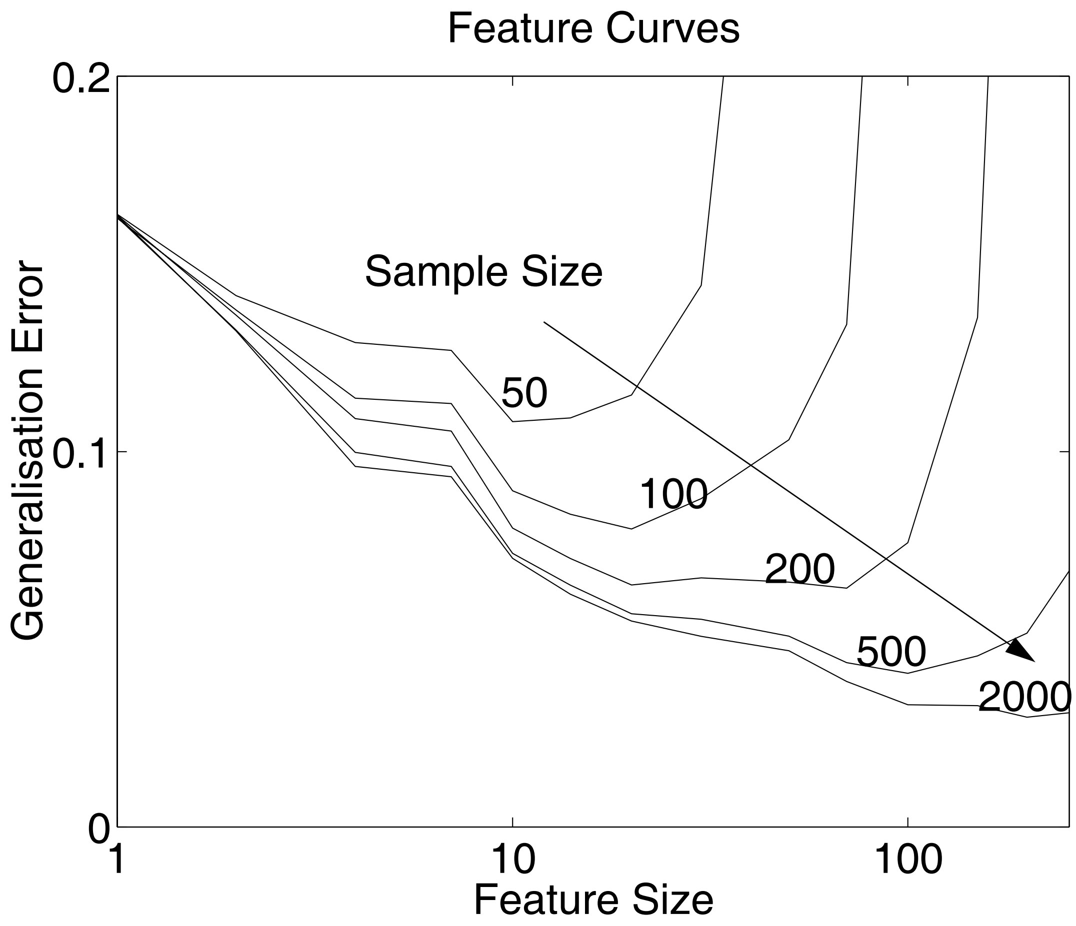
Random Subsets:

- 2 x 1000 Training
- 2 x 1000 Testing

Errors averaged over 50 experiments

# Fisher Results



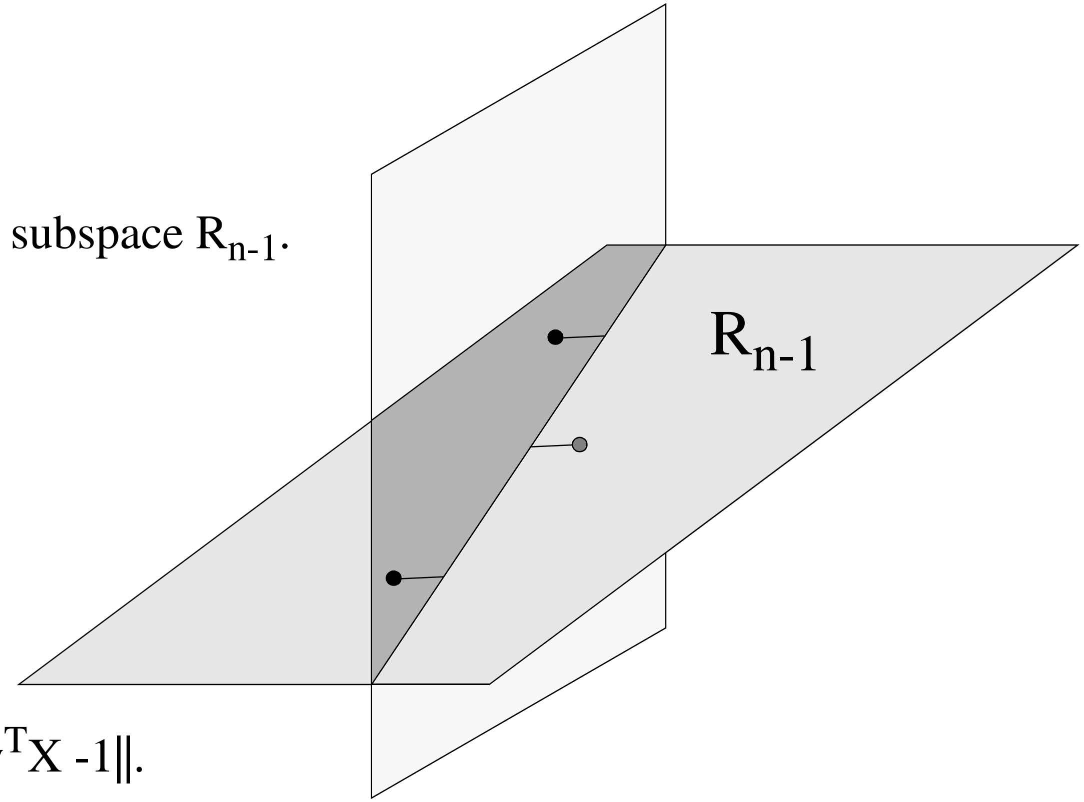




Peaking of the generalization error of FLD as a function of the feature size.

Learning curves of the FLD .

# Pseudo Fisher Linear Discriminant

n points in $R_k$ are in a (n-1) dimensional subspace $R_{n-1}$.



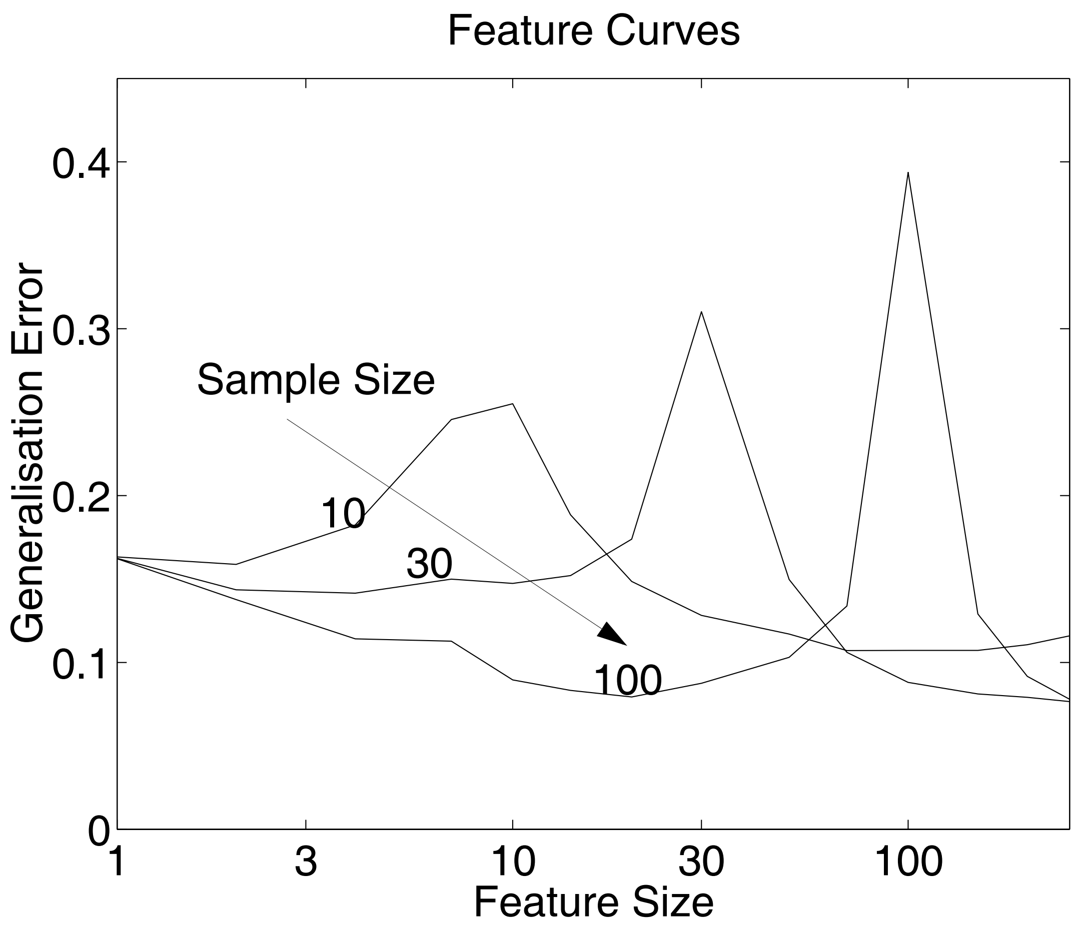

$\rightarrow$ w is the minimum norm solution of $\|w^T X - 1\|$.

$\rightarrow$ Use Moore-Penrose pseudo-inverse: $w^T = \text{pinv}(X)$.

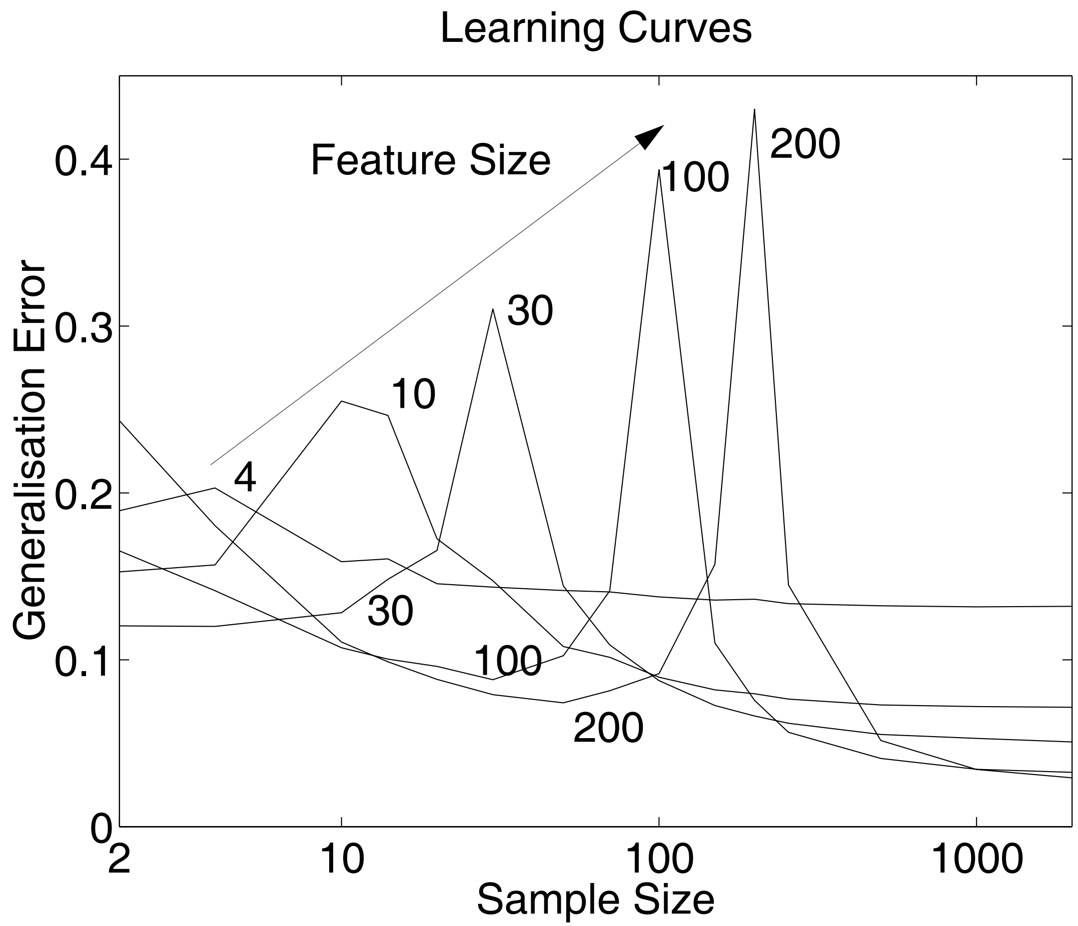
For $n > k$ the same formula defines Fisher's Discriminant.

$X = \{ (x_i, 1), x_i \in A, (-x_j, -1), x_j \in B \}$

# Feature Curves and Learning Curves

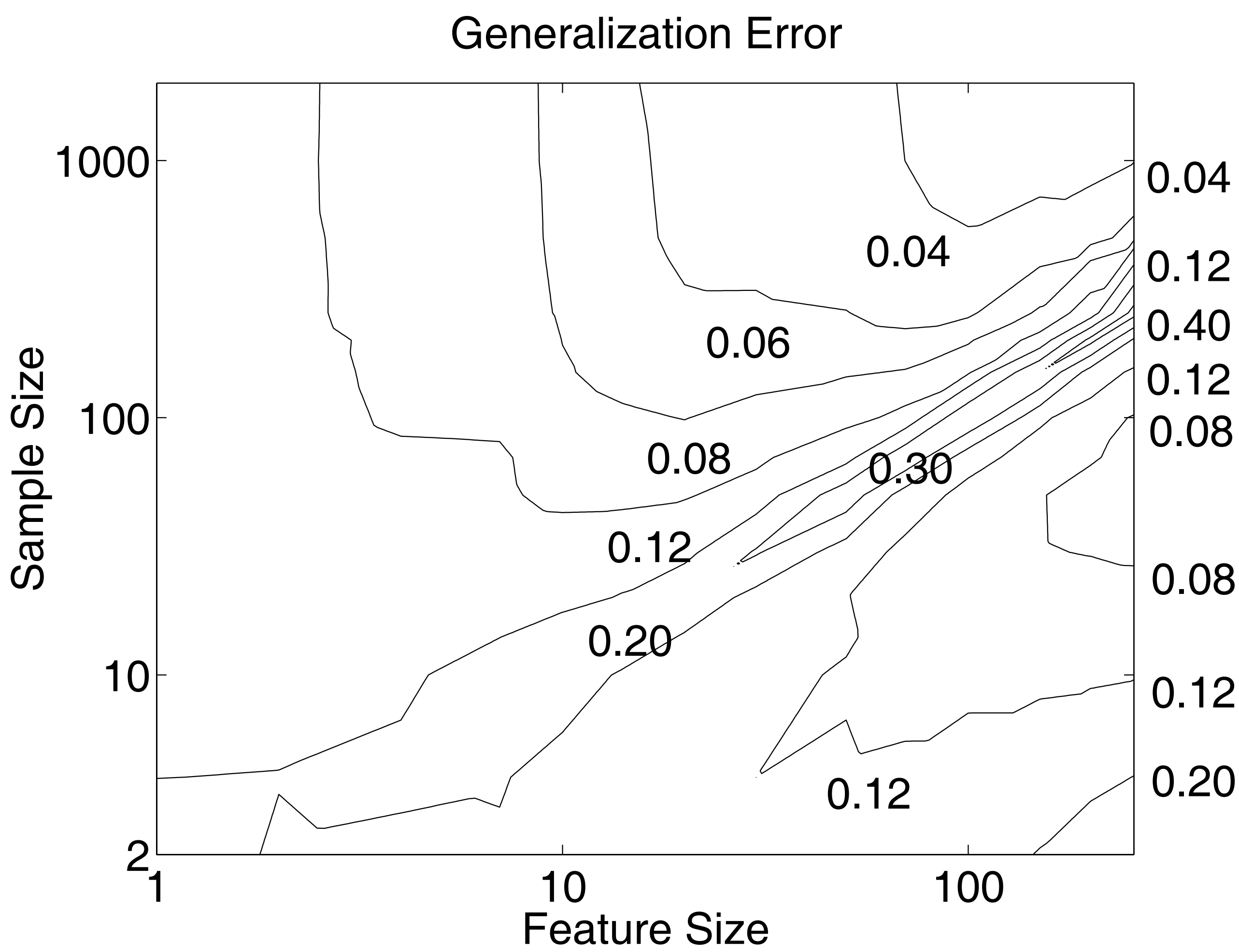



The generalization error of the PFLD as a function of the feature size.




Learning curves of the PFLD.

# Feature Size <--> Sample Size Error




The generalization error of the PFLD as a function of feature size and sample size.

# Improving Pseudo Fisher Linear Discriminant (PFLD)

PFLD is for dimensionalities > sample size fully overtrained.

Still good results are possible ($\varepsilon = 0.08$, 30 objects in 256 D)

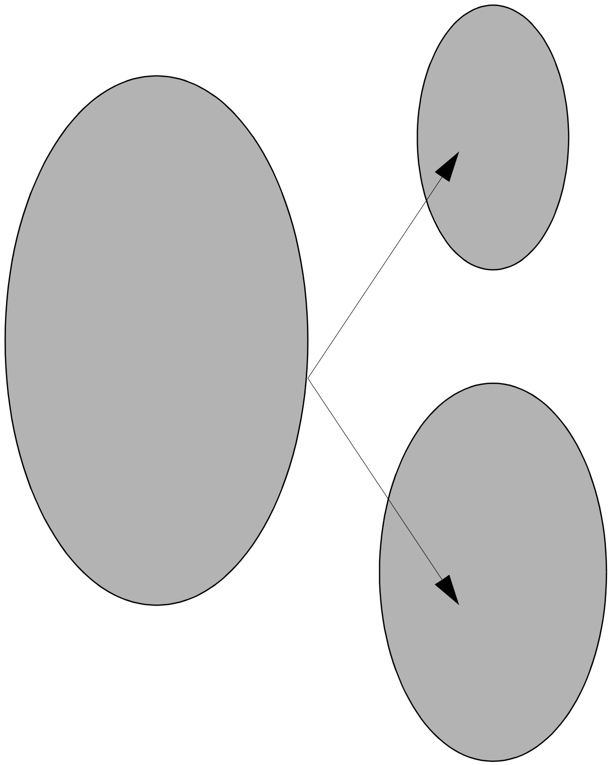
--> Better results are possible

- Regularization e.g. by $D(x) = (\hat{\mu}_A - \hat{\mu}_B)^T (\hat{G} + \lambda I)^{-1} x$

- Change of representation to lower dimensional spaces.

# Representation Sets and Kernel Mapping

Representation Set
$Y = \{\mathbf{y}_i\}, \|Y\| = n$



Training Set $X = \{\mathbf{x}_i\}$
Possibly $Y \subset X$

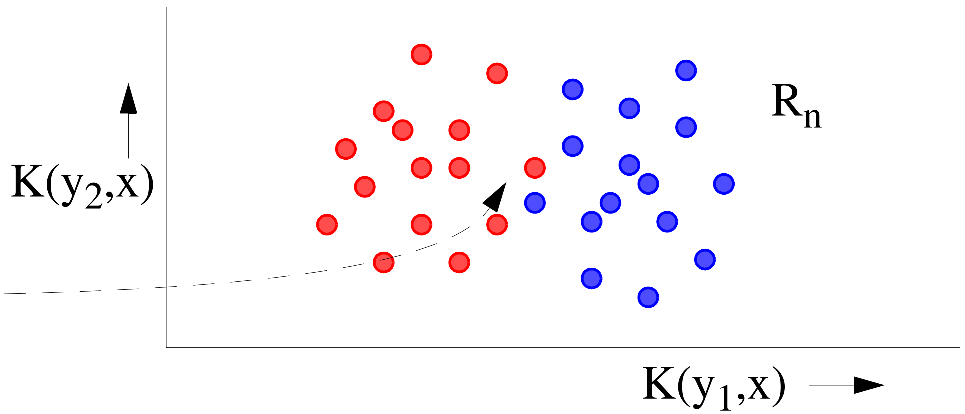
$K = K(Y,\mathbf{x}) = (K(\mathbf{y}_i,\mathbf{x}), i=1, ..., n), \ K \in R_n$

maps an arbitrary object $\mathbf{x}$ into $R_n$

Polynomials: $K(\mathbf{y}_i, \mathbf{x}) = (\mathbf{x} \bullet \mathbf{y}_i + 1)^p$

Gaussians: $K(\mathbf{y}_i, \mathbf{x}) = \exp\left(\dfrac{-\|\mathbf{x} - \mathbf{y}_i\|^2}{2\sigma^2}\right)$

--> Nonlinear Mapping



# Representation Sets

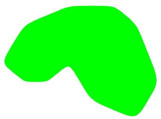
- Dimensionality is controlled by the size of the Representation Set Y.

- Original objects may have arbitrary representation (feature size), just K(**y**,**x**) has to be defined.

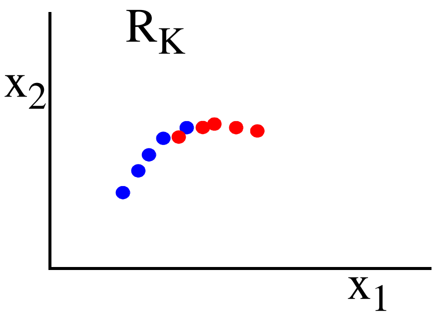
- In the feature space defined by the Representation Set, traditional classifiers may be used.

- Problems: choice of Y, choice of K

# Feature Approach and Representation Sets



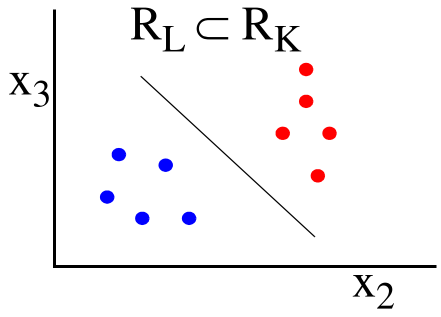
object



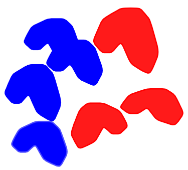

feature space

$$X = \begin{pmatrix} x_{11} & x_{12} \cdots & x_{1k} \\ x_{21} & x_{22} \cdots & x_{2k} \\ x_{31} & x_{32} \cdots & x_{3k} \\ x_{41} & x_{42} \cdots & x_{4k} \end{pmatrix}$$

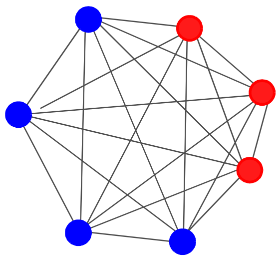
subspace selection




classifier



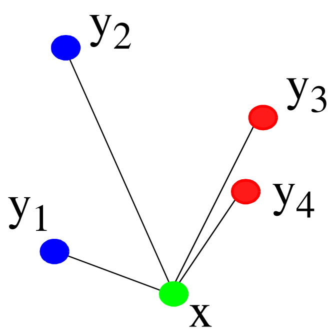
set of objects



relation matrix (kernel K(x,x))

$$\begin{array}{cc} & \quad\quad\quad Y \\ K = & \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{pmatrix} \; X \end{array}$$

selection of Representation Set Y




kernel based classification

TUDelft

# Support Vector Classifier

$$
K = \begin{array}{c} Y \\ \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{pmatrix} \end{array} \quad X
$$

Reduce training set X to

minimum size 'support set' Y

such that

if used as Representation Set,

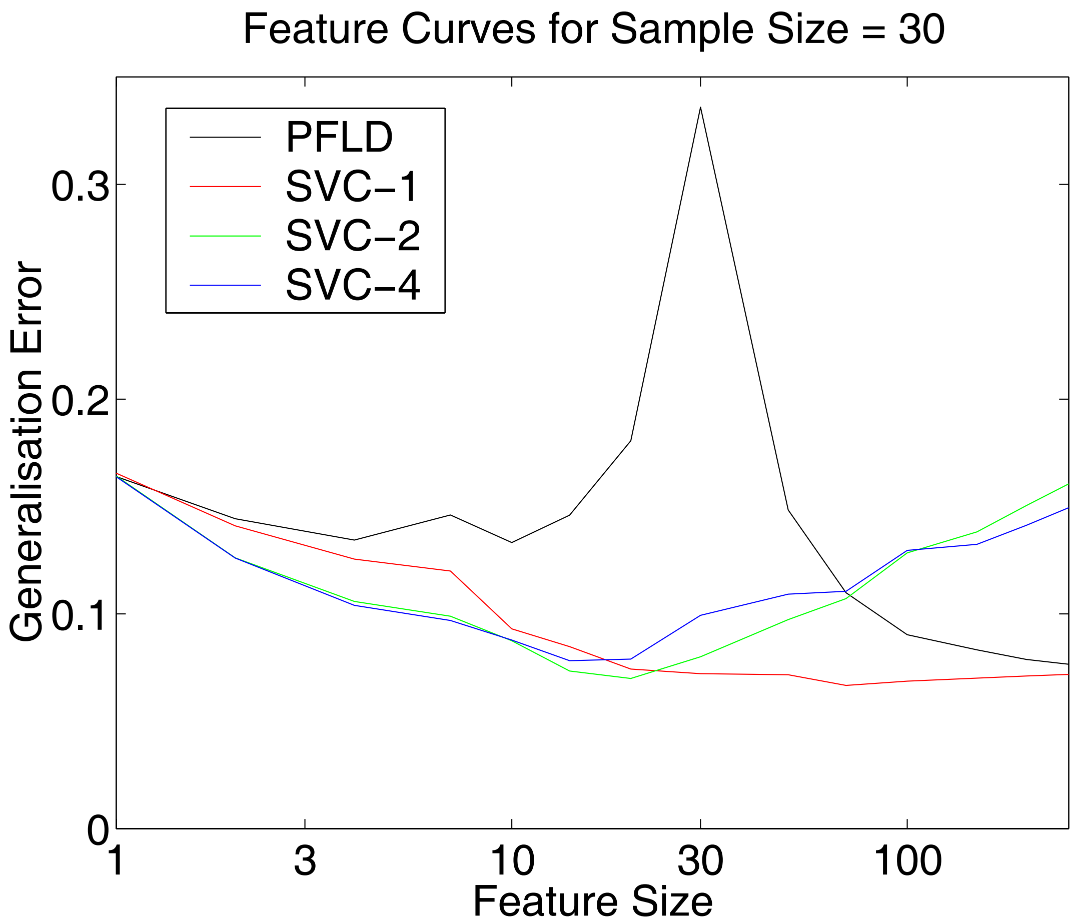
X is error free classified:

$$\min_{\|Y\|} [\text{classf\_error}(K(Y,X))]$$

Notes:

- Classifier is written as a function of n points in $R_n$
- Not all kernels allowed (Mercer's theorem)

# Support Vector Classifier Results




The generalization errors of the PFLD and the SVC as a function of the feature size for a sample size of 30. In the SVC polynomial kernels are used of the orders 1,2 and 4. Number of support vectors: 10 - 30.

# Dissimilarity Based Classification

$$
\begin{array}{ccc}
 & Y & \\
K = & \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{pmatrix} & X
\end{array}
$$

(Random) selection of Representation Set Y.

All objects X are used for training.

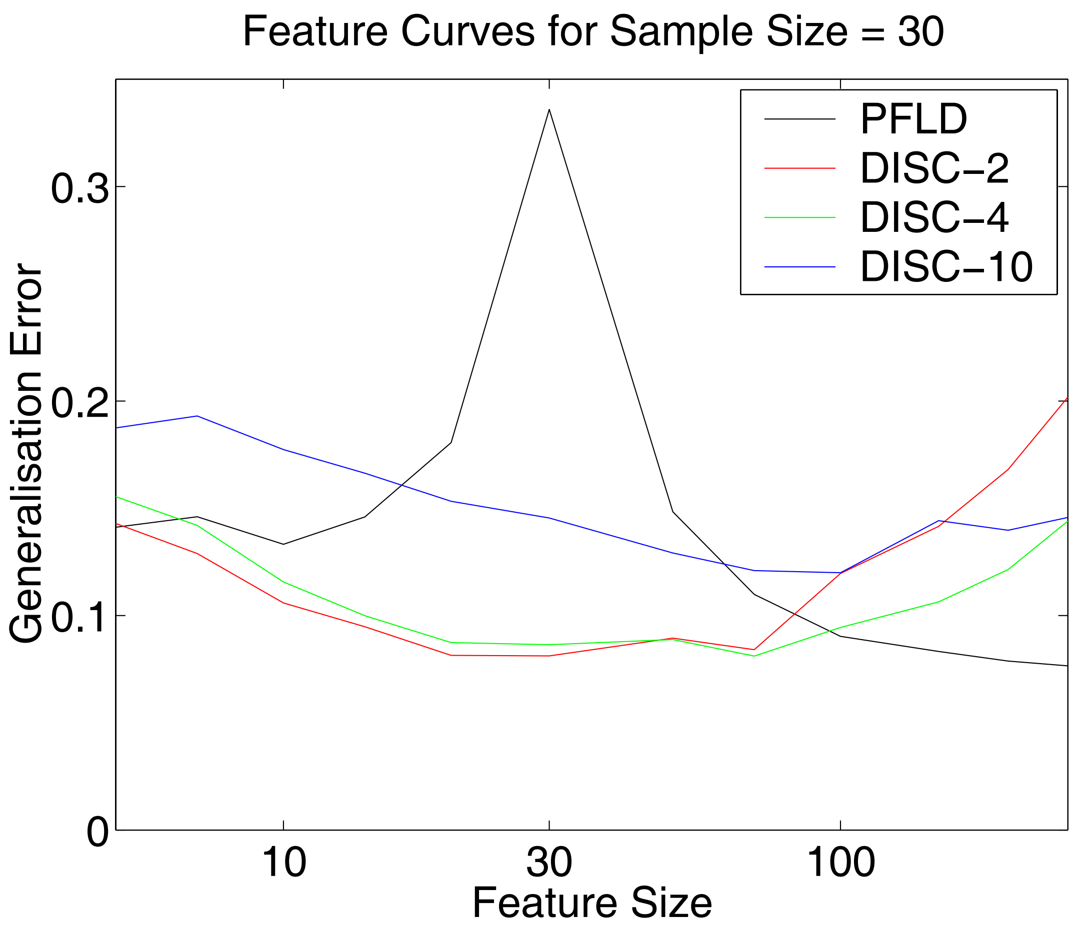
Any kernel $K(\mathbf{y},\mathbf{x})$ is allowed (e.g. $\|\mathbf{x} - \mathbf{y}\|$ )

Fast training (simple selection of Y).

Possibly fast testing (choose small Y).

E. Pekalska et al., Classifiers for dissimilarity-based pattern recognition, ICPR15

# Dissimilarity Based Classification Results




The generalization errors of the PFLD and a linear dissimilarity based classifier (DISC) as a function of the feature size, using a sample size of 30. For DISC three sizes of the representation set are used: 2, 4 and 10.

# Subspace Classifier

$$K = \begin{array}{c} Y \\ \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{pmatrix} \end{array} X$$

K' = PCA(K)

Training Set X equals Representation Set Y.

Dimension reduction per class by PCA.

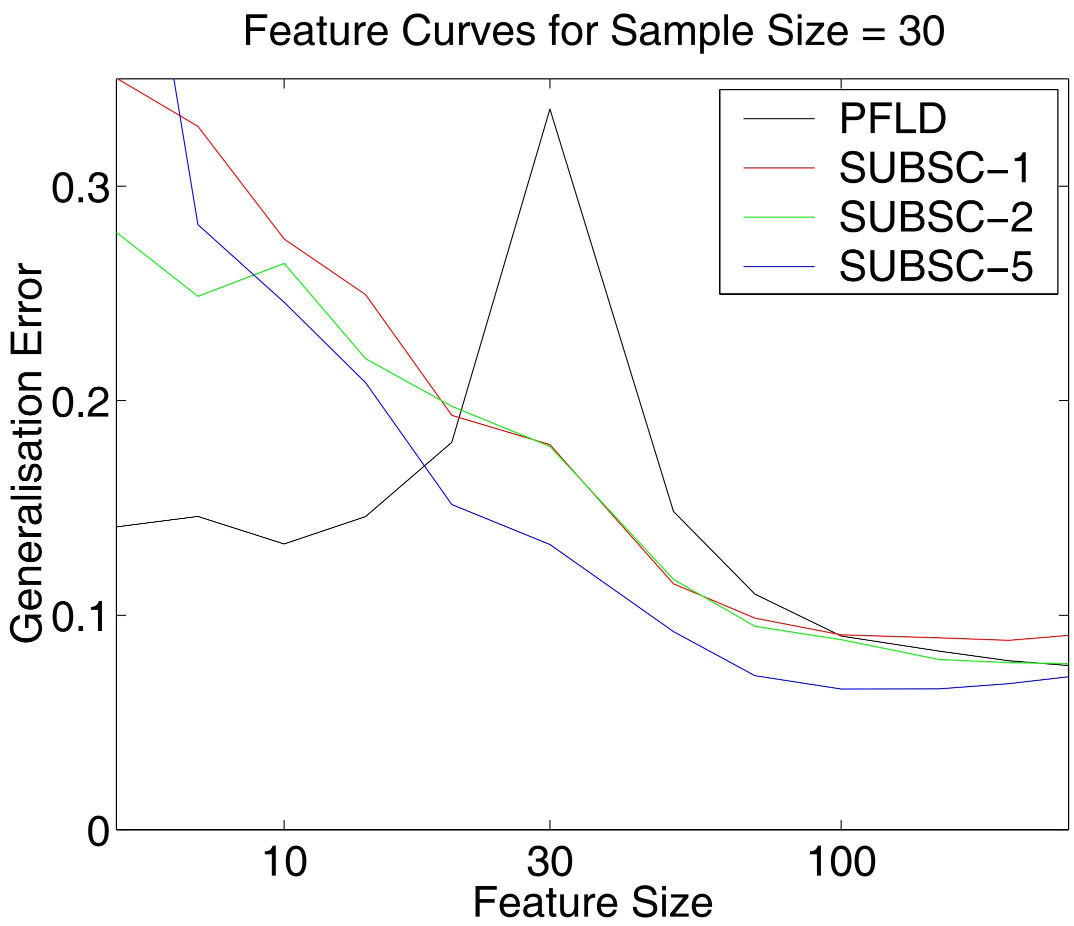
Classification by nearest subspace.

Compare Eigenface method (linear subspace).

Compare feature extraction (no selection).

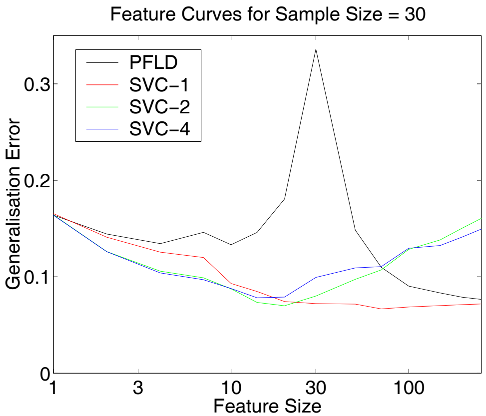
Test objects have to be compared with entire training set (not true for linear inner product kernel).

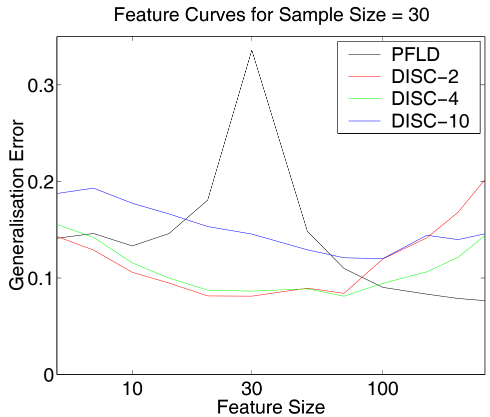
# Subspace Classifier Results

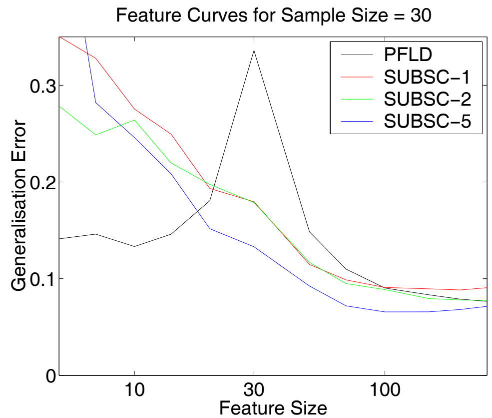



The generalization errors of the PFLD and the subspace classifier (SUBSC) as a function of the feature size for a sample size of 30.
For SUBSC three subspace dimensionalities per class are used: 1, 2 and 5.

# Summary of Almost Empty Space Classifiers



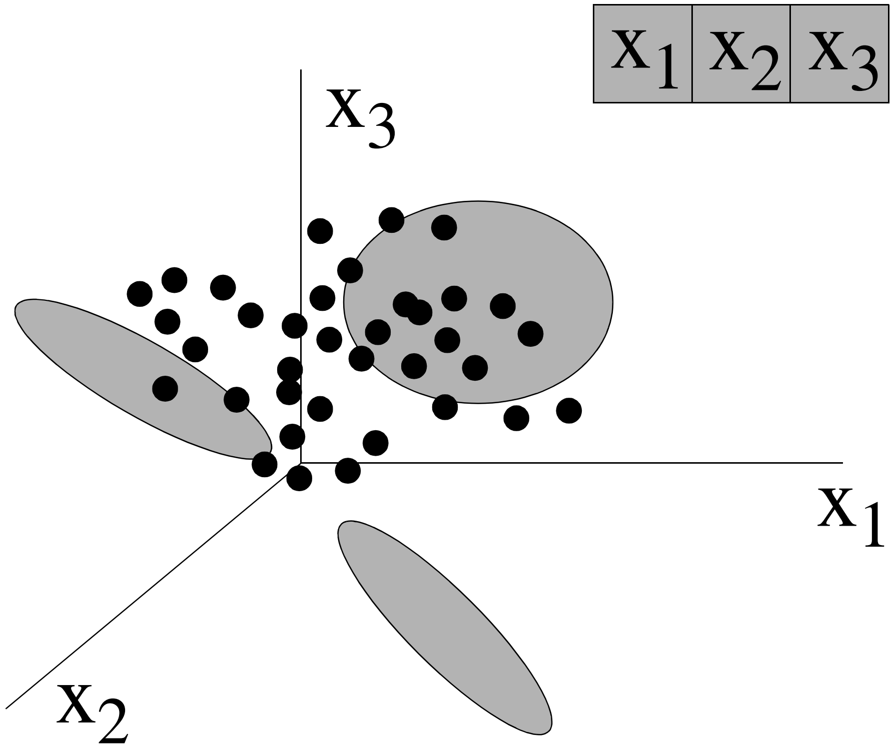




| | Support Vector Classifier | Dissimilarity based Classification | Subspace Classifier |
|---|---|---|---|
| Representation Set Selection | Optimized on Minimum Error | Heuristics Free Choice of n | None |
| Dimension of Representation | n | n | (n=) k |
| Size of Final Training Set | n | k | k |
| Training Effort | high | low | moderate |
| Test Effort | O(n) | O(n) | O(k) |

Training Set X (size k); Representation Set Y (size n); $n < k$ ($n << k$), $Y \subset X$

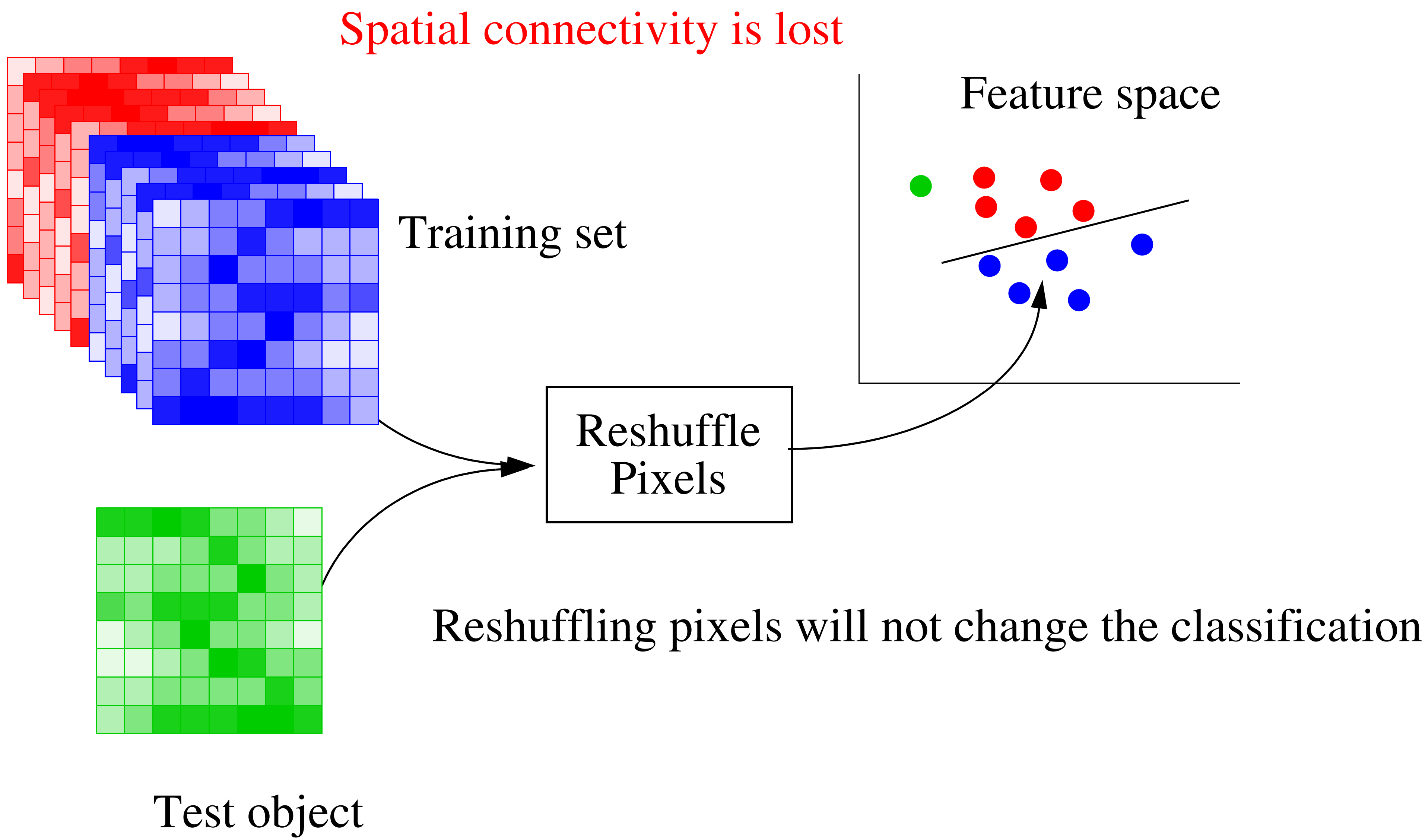
# The Sensor Connectivity Issue

Spatial / temporal / spectral connectivity is lost by
sample based feature space representations



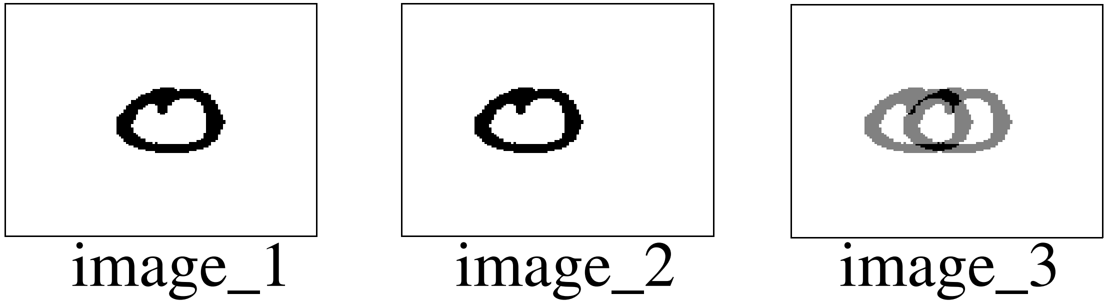

Dependent (connected) measurements are represented independently,
The dependency  has to be refound from the data

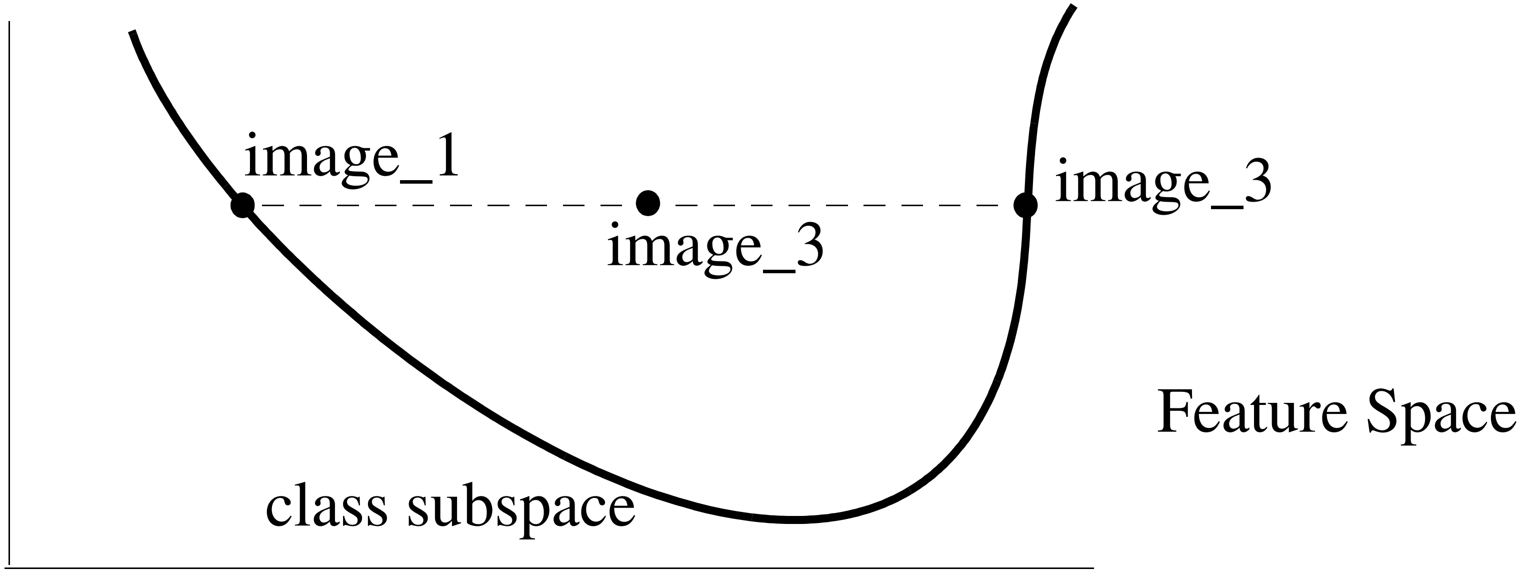
# Problems with the Pixel_Feature Representation



# Problems with the Pixel_Feature Representation

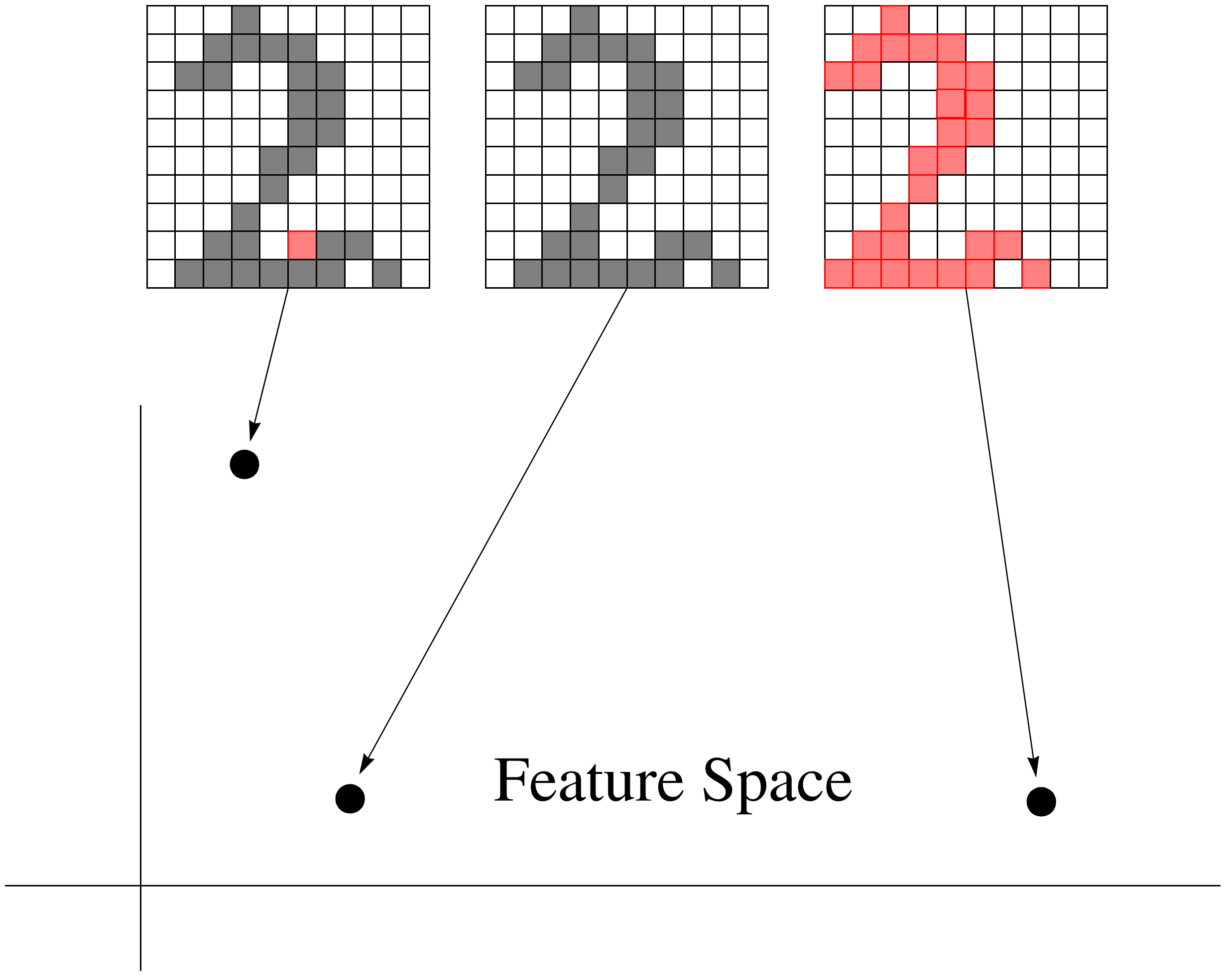
Interpolation does not yield valid objects



# Problems with the Pixel_Feature Representation

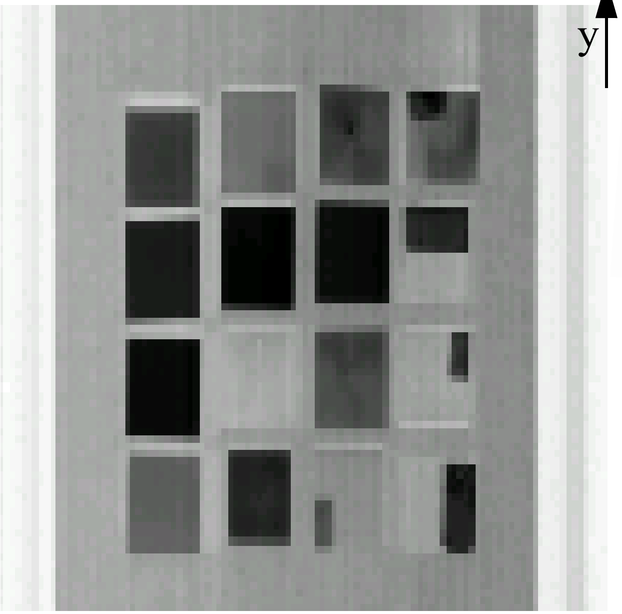
Representation jumps after small disturbances

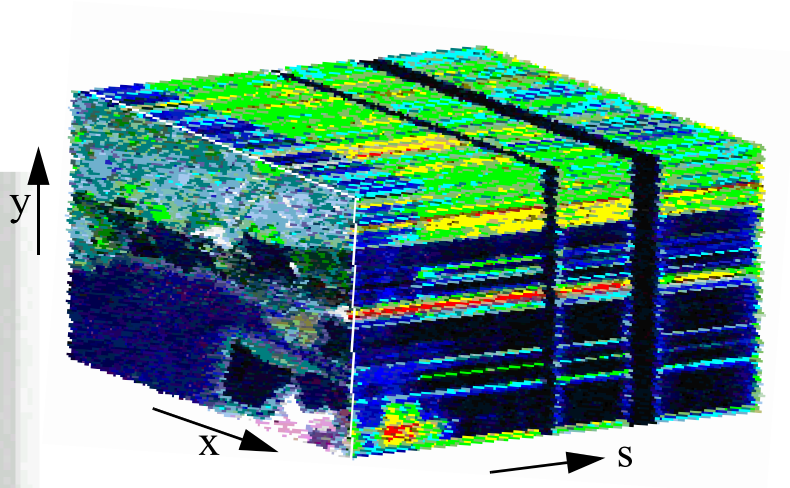


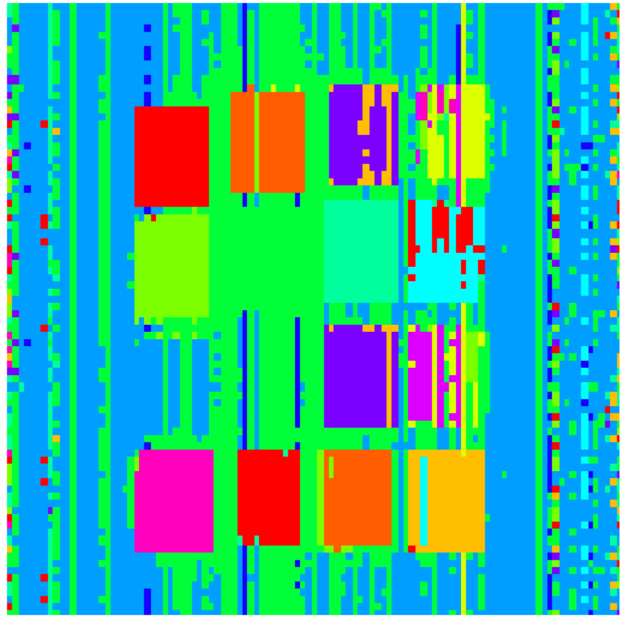
# Connectivity Preserving Representations?

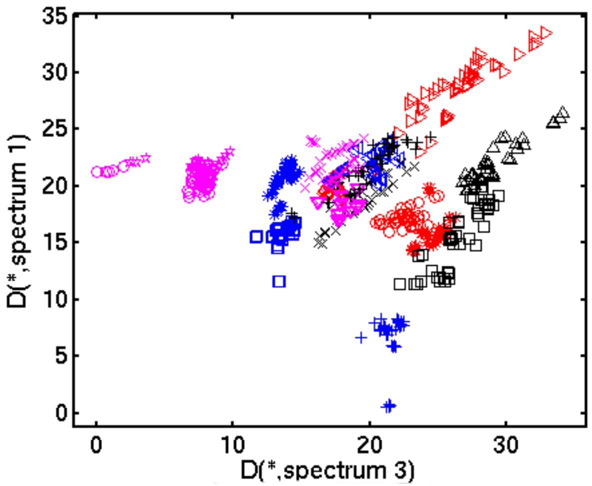
Can we find a basic representation of objects that respects the connectivity between sensor elements (pixels / samples) and that thereby avoids the need to implicetely reconstruct this connectivity from the set of examples?

# Hyperspectral Image Segmentation







# Conclusion

The use of Kernel based Representation Sets allows for the construction of generalizable, nonlinear classifiers in very high-dimensional feature spaces based on relatively small training sets (i.e. size lower than the dimensionality).

This Amost Empty Space Problem might be avoided by a connectivity preserving representation.